Abstract Title Page

Title: Improving Procedural Knowledge and Transfer by Teaching a Shortcut Strategy First

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Abstract Body

Background / Context:

Students often memorize and apply procedures to solve mathematics problems without understanding why these procedures work. In turn, students demonstrate limited ability to transfer strategies to new problem types (see Markovits & Sowder, 1994). Math curriculum reform standards underscore the importance of procedural flexibility and transfer, emphasizing that students need to understand and flexibly adapt strategies when encountering various problem situations (NCTM, 2000; NRC, 2001). Drawing on research in both education and cognitive psychology, the current study examines whether the order in which problem-solving strategies are traditionally taught may result in greater procedural rigidity. Specifically, this work tests the theory that initially instructing students on complex algorithms leads them to rigidly apply these procedures, overlooking more efficient shortcut strategies even when they are later introduced. I examine whether teaching more efficient problem-solving strategies prior to more complex algorithms (i.e., *shortcut-first instruction*) improves procedural flexibility and transfer.

Procedural flexibility and understanding are particularly important in early algebra learning. Algebra is considered a gatekeeper to future educational opportunities (Knuth et al., 2006; Moses & Cobb, 2001; NCTM, 2000). Many researchers and educators maintain that algebra concepts should be taught early in mathematics learning, in order to promote more flexible thinking and corresponding understanding throughout the algebra curriculum (McNeil, 2008; NRC, 2001). One reason for this recommendation is that early math experiences often lead to fundamental misconceptions that inhibit students' ability to think flexibly about problems. For example, both elementary-school and first-year algebra students often struggle with *math equivalence*, the concept that the equal sign represents a relational symbol (Carpenter, Franke, & Levi, 2003; Knuth et al., 2006; NRC, 1998, 2001; RAND Mathematics Study Panel, 2003). Based on extensive early experience solving answer-oriented problems, with the equal sign at the end of the problem (e.g., 5 + 2 = _), students often incorrectly view the equal sign as an operational symbol meaning "the total" or "compute the answer" (Baroody & Ginsburg, 1983; Carpenter et al., 2003; Kieran, 1981; Markovits & Sowder, 1994; McNeil & Alibali, 2005). When encountering problems with operands on both sides of the equal sign (e.g., 5 + 2 = _ + 3), students rigidly apply this operational view of the equal sign, often ignoring the operand on the right side of the equal sign, or adding all the numbers (e.g., 5 + 2 + 3 = 10; Alibali, 1999; McNeil, 2008).

Interventions that target such fundamental concepts early improve students' ability to avoid using incorrect strategies to solve equivalence problems (e.g., McNeil & Alibali, 2005). However, such interventions generally do not target flexibility in applying correct problem-solving solutions. Students often do not demonstrate sufficient understanding to flexibly adapt or transfer procedures used to solve equivalence problems based on the characteristics of the problem at hand (e.g., DeCaro & Rittle-Johnson, 2012; see also Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012). Such flexibility may be important for building reasoning and problem-solving transfer in mathematics more generally, and is critical to algebra learning in particular. If students are expected to be flexible problem-solvers in algebra, then they should practice flexible reasoning in early algebra problem-solving contexts as well (cf. Markovits & Sowder, 1994).

The current study was designed to promote procedural flexibility and transfer in elementary school students' learning of math equivalence, by using shortcut-first instruction. Traditionally, students are taught—and become well-practiced with—complex algorithmic procedures prior to learning shortcut strategies (Hiebert et al., 2003). This study examined the impact of reversing this traditional order of instruction, instructing students on shortcut strategies prior to complex algorithms. Here, *complex algorithms* refer to procedures that require many steps, but can be used to solve all problems in the domain. For example, to solve math equivalence problems (e.g., 5 + 2 + 3 = _ + 3), students are often taught the *add-subtract* strategy—add up the values on the left side of the equal sign and subtract the value(s) on the right side. Students are rarely taught *shortcut strategies* that require fewer computational steps, but cannot necessarily be applied to all problems. For example, the *grouping* strategy may be used when both sides of the problem have a repeated addend (e.g., the "3" in the problem 5 + 2 + 3 = _ + 3). Use of this procedure requires eliminating the repeated addend and solving the problem without these values (e.g., 5 + 2 = _). Despite the efficiency of grouping, students apply this procedure quite infrequently (e.g., DeCaro & Rittle-Johnson, 2012).

Research in cognitive psychology and education suggests that teaching students a shortcut strategy first may reduce students' tendency to rely solely on complex algorithms and may improve their ability to adapt and transfer strategies to the problem context. Specifically, evidence suggests that students' strategy choices are based primarily on associative processes, rather than conscious, deliberate choices about which strategy to select on every given problem (Siegler & Jenkins, 1989; Verschaffel et al., 2009). Strategies that have been used most often in the past are most likely to be implemented on subsequent problems, as they gain associative strength (Shrager & Siegler, 1998; Siegler & Stern, 1998). However, students are thought to monitor these associative-based strategies via metacognitive processes and can override these default strategy choices to instead rely on deliberative strategies when aspects of a problem-solving context indicate the need to do so (Verschaffel et al., 2009).

If traditional instructional practice is to teach, and have students practice, a complex algorithm to solve a given problem type (e.g., the add-subtract strategy for math equivalence problems), then the associative strategy choice will be that complex algorithm. Students will be most inclined to use the complex algorithm, even when more efficient strategies are available. Thus, students are likely to demonstrate *entrenchment* in the previously-learned algorithm. Entrenchment occurs when students become well-practiced with certain patterns or mental representations within a domain, and they persist in applying these representations even when they are less optimal (McNeil, 2007; McNeil & Alibali, 2005; Ricks, Turley-Ames, & Wiley, 2007).

One way to reduce entrenchment in complex algorithms may be to introduce shortcut strategies first. If students learn and practice more efficient shortcut strategies prior to learning complex algorithms, their use of the shortcut increases in associative strength (cf. Siegler, 2002). Thus, when encountering new problems, the default strategy choice will likely be the shortcut strategy. By definition, the shortcut strategy can be implemented with greater efficiency, but not all problems can be solved with this procedure. For example, in the domain of math equivalence, the grouping strategy can be applied to many problems, but not all (i.e., those without a repeated addend). A more complex algorithm will be necessary to solve the latter type of problem. In shortcut-first instruction, students are instructed on this algorithm only after instruction and practice with the shortcut strategy. Students therefore have the algorithm in their procedural repertoire, but should more automatically implement the shortcut before overriding this default strategy if necessary. Moreover, because the shortcut strategy requires students to attend to important problem features (e.g., the presence of repeated addends), students may be better able to notice when these features are not present in a new problem, facilitating adaptive strategy selection. Indeed, students' ability to use information about problem features may improve their transfer to new problem types, because they may better understand when certain procedures are best used (cf. McNeil & Alibali, 2004).

Purpose / Objective / Research Question / Focus of Study:
The current study was designed to provide an initial test of the hypothesis that shortcut-first instruction will improve learning outcomes, compared to the traditional method of instructing students on shortcuts following complex algorithms. It was expected that shortcut-first instruction would improve learning across a variety of learning outcomes, particularly procedural flexibility and transfer.

Setting:
The study was conducted in two private schools in a large urban area.

Population / Participants / Subjects:
Participants were 54 second- and third-grade children who scored below 50% on a pretest measuring knowledge of math equivalence (age *M* = 8.05 years, *SD* = .51; 74% female, 15% ethnic minorities). Data from 13 additional children were excluded due to experimenter error (*n* = 1), unavailability for the posttest or retention test (*n* = 3) or teacher-reported learning disabilities (*n* = 13).

Intervention / Program / Practice:
In individual tutoring sessions (15 min), students were randomly assigned to one of two conditions, beginning with a brief lesson on the concept of math equivalence. Then, students in the intervention condition (*shortcut-first condition*) were taught, and practiced, a shortcut strategy (grouping), followed by a complex algorithm (add-subtract). Students in the control condition (*complex-first condition*) completed the same activities in the opposite order: receiving instruction and practice on the complex algorithm, then the shortcut. Following instruction, all students were reminded of both strategies and solved a final set of problems. Each practice set included four math equivalence problems with repeated-addends (e.g., 3 + 4 + 5 = _ + 5).

Research Design:
A pretest – intervention – immediate posttest – delayed retention test design was used. Students were randomly assigned to one of two between-subjects conditions.

Data Collection and Analysis:
Students completed a pretest in their classrooms (30 min), and those selected for the study completed an individual tutoring intervention and immediate posttest within one week (60 min). Two weeks later, students completed a retention test in their classrooms (30 min). A previously validated measure of math equivalence (Rittle-Johnson et al., 2011) was used at each time point. The pretest included two subscales: procedural knowledge (9 items) and conceptual knowledge (10 items). The posttest and retention test added measures of transfer (3 items; shortened) and procedural flexibility (created based on previous work; e.g., Rittle-Johnson, Star, & Durkin, 2012). Example items are presented in Table 1.

At pretest, there were no differences based on condition for procedural knowledge ($M = 2.3\%$, $SD = 0.74\%$) or conceptual knowledge ($M = 20.6\%$, $SD = 12.8\%$), Fs < 1. Posttest and retention test data were analyzed using 2 (condition: shortcut-first, complex-first) x 2 (time of test: posttest, retention test) mixed-factorial ANCOVAs for each subscale, with pretest scores as covariates.

Findings / Results:

No significant effects of time of test were found; therefore, results are reported as averaged across posttest and retention test. As shown in Figure 1, students in the shortcut-first condition outperformed students in the complex-first condition on two subscales: procedural knowledge, $F(1,50) = 4.82$, $p = .033$, $\eta_p^2 = .09$, and transfer, $F(1,50) = 4.63$, $p = .036$, $\eta_p^2 = .09$. This pattern did not reach significance on the conceptual knowledge or procedural flexibility subscales, Fs < 1. However, students in the shortcut-first condition used a greater number of different correct strategies to solve the procedural knowledge items ($M = 1.36$ out of 3, $SE = .11$) than students in the complex-first condition ($M = .98$, $SE = .12$), $F(1,36) = 4.99$, $p = .032$, $\eta_p^2 = .12$, indicating that they adopted multiple strategies to solve the problems overall.

Conclusions:

Although students in both conditions completed the exact same learning activities, providing instruction on shortcut strategies first improved students' ability to solve math equivalence problems and transfer concepts and procedures to solve novel, difficult problems. Students also demonstrated greater procedural flexibility, when measured as greater use of multiple, correct strategies. However, differences between conditions were not found on a separate procedural flexibility subscale, or on a measure of conceptual knowledge. Taken together, these findings provide strong initial support for the proposed theory and use of a shortcut-first intervention, and stand in stark contrast to the traditional method of instructing students on complex strategies first.

Other interventions that target procedural flexibility and transfer generally place a significant burden on cognitive resources, and it is only students with higher ability (Verschaffel et al., 2009) or higher prior knowledge (Rittle-Johnson et al., 2009) who can benefit. Shortcut-first instruction may benefit learning by reducing this cognitive load—capitalizing on associative learning processes to help students learn and adapt strategies. Future research should examine whether this intervention may be especially useful for students with lower ability, who would otherwise be at a disadvantage in learning multiple strategies.

Although promising, the current results are limited to an initial test of a novel mathematics intervention developed based on research in both education and cognitive psychology. More work is needed to substantiate the mechanisms by which learning is improved and determine whether this intervention can be extended to classroom instruction. Future research is also needed to determine if the benefits of this method of supporting procedural knowledge, flexibility, and transfer in early algebra learning extends to more complex algebra learning in later school years.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Table 1
Example Items from the Math Equivalence Assessment

Subscale	Example Item	Scoring Criteria
Procedural Knowledge	3 + 7 + 6 = 3 + ☐	Use correct strategy (must be ±1 of correct answer)
Conceptual Knowledge	What does the equal sign mean?	Provide relational definition (e.g., the same amount as)
	Indicate whether equations such as 3 = 3 are true or false	Correctly recognize nonstandard equations as true or false
Transfer	64 + 5 + 8 = ☐ + 8	Use correct strategy (must be ±1 of correct answer)
Procedural Flexibility	Find the number that goes in each box, using two different ways of getting the answer.	Provide 2 correct, unique solutions.

Figure 1
Learning Outcomes as a Function of Intervention Condition